No Sommerfeld resummation factor in $e^+e^- \to p\overline{p}$?

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ABSTRACT: The Sommerfeld rescattering formula is compared to the $e^+e^- \to p\overline{p}$ *BABAR* data at threshold and above. While there is the expected Coulomb enhancement at threshold, two unexpected outcomes have been found: $|G^p(4M_p^2)| = 1$, like for a pointlike fermion, and moreover data show that the resummation factor in the Sommerfeld formula is not needed. Other $e^+e^- \to$ baryon-antibaryon cross sections show a similar behavior near threshold.

Many recent papers, mostly concerning evidence of Dark Matter [1], are related to the so called Sommerfeld rescattering formula [2] [eq. (2)]. In this letter the unexpected lack of the resummation term in the Sommerfeld rescattering formula in the present $e^+e^- \to p\overline{p}$ cross section data, as well as other unexpected features of $e^+e^- \to \mathcal{B}\overline{\mathcal{B}}$ ($\mathcal{B}$ stands for baryon), are emphasized, namely:

- in $e^+e^- \to p\overline{p}$ the cross section is not vanishing at threshold, as already pointed out [3, 4], fully dominated by the Coulomb final state enhancement and $|G^p(4M_p^2)| = 1$, where G^p is the common value of electric and magnetic proton form factors at the production threshold;

- in $e^+e^- \to p\overline{p}$ the cross section above threshold is consistent with no Sommerfeld resummation factor, as it will be shown in detail in the following;

- other charged baryon pair cross sections data, $e^+e^- \to \Lambda_c\overline{\Lambda_c}$ and $e^+e^- \to p\overline{N}(1440) +$ c.c., show similar features, even if within large errors, as well as the present puzzling data on neutral baryon pair cross sections.

These cross sections have been measured by means of initial state radiation. This technique has several advantages in the case of pair production; in particular at threshold in the pair center of mass (c.m.) energy:

- the efficiency is quite high;

- a very good invariant mass resolution is achieved, $\Delta W_{p\overline{p}} \sim 1$ MeV, comparable to what is obtained in a symmetric storage ring.

In Born approximation the cross section for the process $e^+e^- \to \mathcal{B}\overline{\mathcal{B}}$ in the case of charged baryon pairs is assumed to be:

$$\sigma_{\mathcal{B}\overline{\mathcal{B}}}(W_{\mathcal{B}\overline{\mathcal{B}}}^2) = \frac{4\pi\alpha^2\beta}{3W_{\mathcal{B}\overline{\mathcal{B}}}^2} \, \mathcal{C} \left[\left|G_M^{\mathcal{B}}(W_{\mathcal{B}\overline{\mathcal{B}}}^2)\right|^2 + \frac{2M_{\mathcal{B}}^2}{W_{\mathcal{B}\overline{\mathcal{B}}}^2} \left|G_E^{\mathcal{B}}(W_{\mathcal{B}\overline{\mathcal{B}}}^2)\right|^2 \right] , \tag{1}$$

where $W_{\mathcal{B}\overline{\mathcal{B}}}$ is the $\mathcal{B}\overline{\mathcal{B}}$ invariant mass, β is the velocity of the outgoing baryon, $\mathcal{C}$ is the Coulomb factor

$$\mathcal{C} = \frac{\pi\alpha/\beta}{1 - \exp(-\pi\alpha/\beta)} , \tag{2}$$

that takes into account the electromagnetic $\mathcal{B}\overline{\mathcal{B}}$ final state interaction, $G_M^{\mathcal{B}}$ and $G_E^{\mathcal{B}}$ are the magnetic and electric Sachs form factors (FF).

Because of parity conservation S and D wave only are allowed. At threshold it is assumed that, according to the analyticity of the Dirac and Pauli FF's as well as according to the S-wave dominance, there is only one FF: $G_E^{\mathcal{B}}(4M_{\mathcal{B}}^2) = G_M^{\mathcal{B}}(4M_{\mathcal{B}}^2) \equiv G^{\mathcal{B}}(4M_{\mathcal{B}}^2)$. The relationship between $G_E^{\mathcal{B}}$, $G_M^{\mathcal{B}}$ and $G_S^{\mathcal{B}}$, $G_D^{\mathcal{B}}$, the partial-wave FF's, are:

$$G_S^{\mathcal{B}} = \frac{2G_M^{\mathcal{B}}\sqrt{W_{\mathcal{B}\overline{\mathcal{B}}}^2/4M_{\mathcal{B}}^2} + G_E^{\mathcal{B}}}{3} , \quad G_D^{\mathcal{B}} = \frac{G_M^{\mathcal{B}}\sqrt{W_{\mathcal{B}\overline{\mathcal{B}}}^2/4M_{\mathcal{B}}^2} - G_E^{\mathcal{B}}}{3} . \tag{3}$$

and $G_D^{\mathcal{B}}(4M_{\mathcal{B}}^2) = 0$. For pointlike fermions it is $G_E(Q^2) = G_M(Q^2) \equiv 1$.

The Coulomb factor, $\mathcal{C}$, is usually introduced as an enhancement factor $\mathcal{E}$ times a resumma-tion term $\mathcal{R}$, i.e. the so called Sommerfeld-Schwinger-Sakharov rescattering formula [2, 5]: $\mathcal{C} = \mathcal{E} \times \mathcal{R}$. It has a very weak dependence on the fermion pair total spin and corresponds to the squared value of the Coulomb scattering wave function at the origin, assumed as a good approximation of the Coulomb final state interaction. In such an approximation the factor $\mathcal{C}$ should affect the S wave only, because the D wave vanishes at the origin. For the same reason a Coulomb enhancement is not expected when pseudoscalar meson pairs are produced via e^+e^- annihilation; in fact these processes occur only in P wave. The cross section formula should be more properly written in terms of S and D wave FF's:

$$\sigma_{\mathcal{B}\overline{\mathcal{B}}}(W_{\mathcal{B}\overline{\mathcal{B}}}^2) = 2\pi\alpha^2\beta \frac{4M_{\mathcal{B}}^2}{W_{\mathcal{B}\overline{\mathcal{B}}}^4}\left[\mathcal{C}\,|G_S^{\mathcal{B}}(W_{\mathcal{B}\overline{\mathcal{B}}}^2)|^2 + 2|G_D^{\mathcal{B}}(W_{\mathcal{B}\overline{\mathcal{B}}}^2)|^2\right]. \tag{4}$$
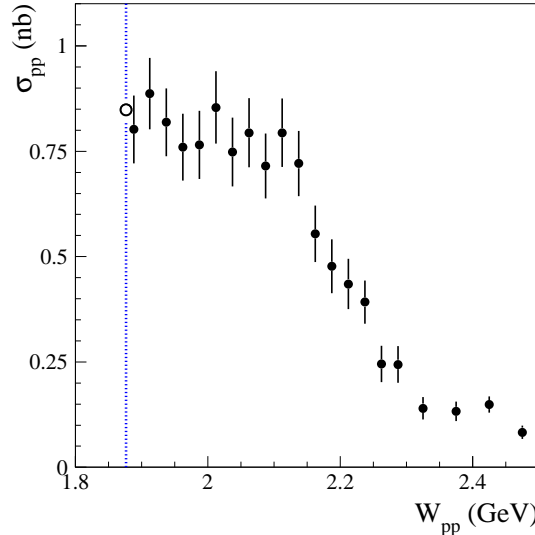
The enhancement factor is

$$\mathcal{E} = \frac{\pi\alpha}{\beta},$$

so that the phase space factor β is cancelled and the cross section is expected to be finite and not vanishing even exactly at threshold. As the resummation factor is

$$\mathcal{R} = \frac{1}{1 - \exp\left(-\pi\alpha/\beta\right)}, \tag{5}$$

it follows that few MeV above the threshold it is $\mathcal{C} \sim 1$, the phase space factor is restored and Coulomb effects can be neglected.

Figure 1: The $p\overline{p}$ cross section obtained by *BABAR* (solid circles). The dotted line represents the threshold and the empty circle is the Coulomb-enhanced threshold cross section, assuming $|G^p(4M_p^2)| = 1$.

Concerning P and D waves a further degree of approximation could be applied by means

of the derivative at the origin or by means of a different approach [6].

The $e^+e^- \to p\overline{p}$ cross section [7] in Fig. 1 shows the following peculiar features:

- it is suddenly different from zero at threshold, being 0.85 ± 0.05 nb (by the way it is the only endothermic process that has shown this peculiarity);

- it is flat above threshold, within the experimental errors, in a c.m. energy interval of about 200 MeV and then drops abruptly.

The expected Coulomb-corrected cross section at threshold is [see eq. (1)]

$$\sigma_{p\overline{p}}(4M_p^2) = \frac{\pi^2\alpha^3}{2M_p^2} \cdot |G^p(4M_p^2)|^2 = 0.85 \cdot |G^p(4M_p^2)|^2 \text{ nb},$$

in striking similarity with the measured one, if $|G^p(4M_p^2)| = 1.00 \pm 0.05$, and Coulomb interaction dominates the energy region at threshold.

Above threshold $|G_S^p|$ and $|G_D^p|$ have already been achieved [8], according to eq. (3) by means of the proton angular distribution and using a dispersion relation procedure applied to space-like and time-like data on the ratio $|G_E^p/G_M^p|$. Dispersion relations are needed to have access to the relative phase between G_S^p and G_D^p. In the c.m. energy range where the cross section is flat, real G_S^p and G_D^p, as well as G_D^p negative came out. For the purposes of this letter this result is kept and G_S^p and G_D^p are reevaluated more exactly according to eq. (4) [however the outcome is essentially the same achieved according to eq. (1)].
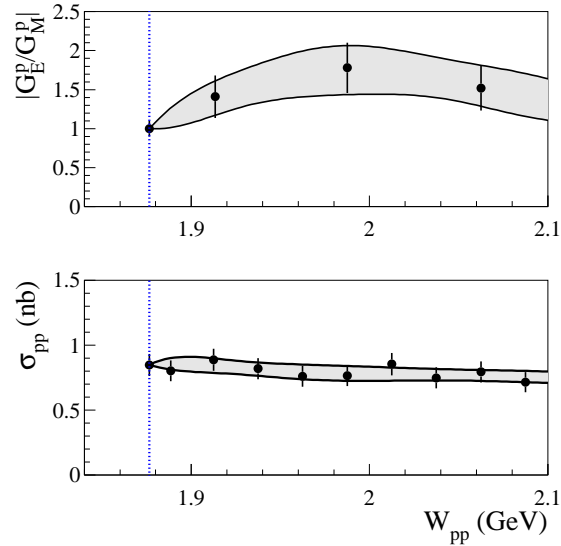
Figure 2: Top figure: modulus of the ratio $|G_E^p/G_M^p|$; bottom figure: $p\overline{p}$ cross section. The gray bands are the fits and the dotted line is the threshold.

Figure 2 shows the near-threshold data on the ratio $|G_E^p/G_M^p|$ and $p\overline{p}$ total cross section that have been used to obtain new $|G_S^p|$ and $|G_D^p|$, as shown in fig. 3, where $|G_S^p|$ is compared

to the inverse of the square root of the resummation factor of eq. (5) and it turns out

$$|G_S^p(4M_p^2 \div 4\,\text{GeV}^2)| \simeq \frac{1}{\sqrt{\mathcal{R}}} = \sqrt{1 - \exp(-\pi\alpha/\beta)}\,.$$

The agreement, within the errors, is striking. In other words: if the resummation factor is introduced in the Coulomb factor then the inverse of the resummation factor is demanded in $|G_S^p|^2$ by the data, strongly suggesting that it is an unnecessary factor. If the resummation factor is not taken into account, it is $|G_S^p| \sim 1$ in a ~ 200 MeV c.m. energy interval above threshold and then it drops abruptly.

This conclusion could have been already foreseen on the basis of the flat cross section above threshold, unrelated to the expected steep increase above threshold due to the phase space. There must be a cutoff to the Coulomb dominance, but, what proton data are showing is that the energy scale for a baryon pair is hundred times greater than that expected for pointlike charged fermions.
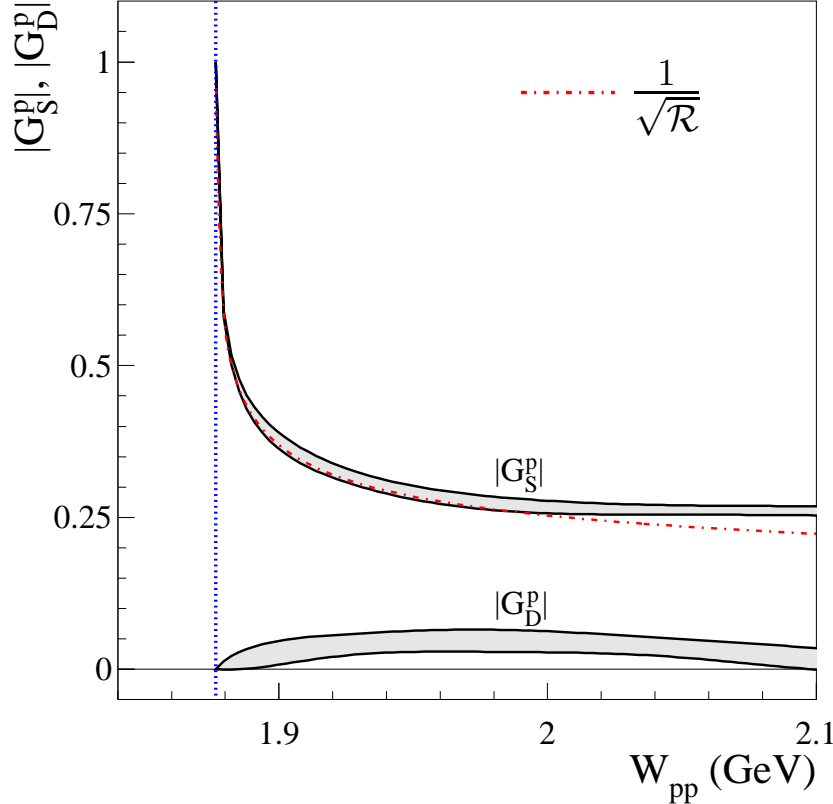

Figure 3: $|G_S^p|$ and $|G_D^p|$ obtained using the ratio $|G_E/G_M|$, the total $p\overline{p}$ cross section, and assuming a relative phase $\phi = \pi$, see text. The dot-dashed curve is the inverse of the square root of the resummation factor of eq. (5).

Also in the case of $e^+e^- \to \Lambda_c\overline{\Lambda_c}$, see fig. 4, as already pointed out [4], the cross section measured by the Belle Collaboration [9] is not vanishing at threshold. If there is no resummation factor there is no major dependence on the mass resolution and the expected cross

section at threshold can be directly compared to the data, once the Coulomb enhancement is taken into account as well as assuming $|G^{\Lambda_c}(4M_{\Lambda_c}^2)| = 1$. There is a fair agreement, within the errors.

Measuring $e^+e^- \to p\overline{p}$ *BABAR* has also measured the cross section of $e^+e^- \to p\overline{N}(1440) +$ c.c., being a significant background to $e^+e^- \to p\overline{p}$. To get from these data a cross section at threshold, a procedure has been exploited to avoid $N(1440)$ finite-width effects. The $N(1440)$ width as well as the $e^+e^- \to p\overline{N}(1440) +$ c.c. are simulated. For each simulated event the $N(1440)$ momentum is evaluated and a new c.m. energy is achieved assuming a zero width. The cross section obtained in this way is compared in fig. 5 to the pointlike cross section, Coulomb enhanced at threshold. Again there is agreement, suggesting that at threshold baryon pair production cross section behaves in a universal way.

At last in the case of $e^+e^- \to \Lambda\overline{\Lambda}$, being Λ a neutral baryon, final state Coulomb effects should not be taken into account and a finite cross section at threshold is not expected. Nevertheless the $e^+e^- \to \Lambda\overline{\Lambda}$ cross section data (Fig. 6) show a threshold behavior similar to that of $\sigma_{p\overline{p}}$.
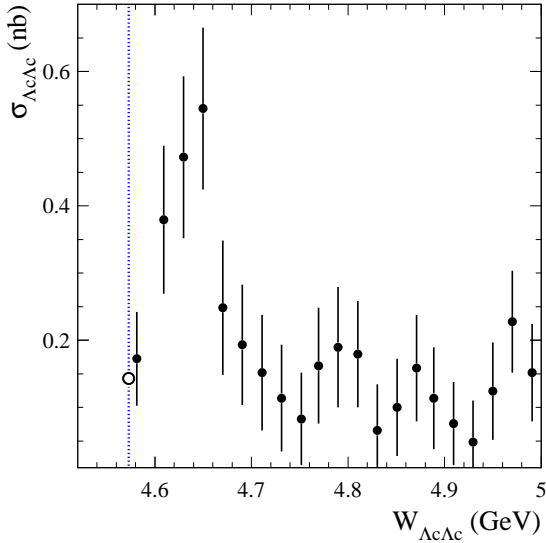
Figure 4: The $\Lambda_c\overline{\Lambda_c}$ cross section obtained by Belle (solid circles). The dotted line represents the threshold and the empty circle is the cross section value Coulomb enhanced at threshold, assuming $|G_c^{\Lambda}(4M_{\Lambda_c}^2)| = 1$.
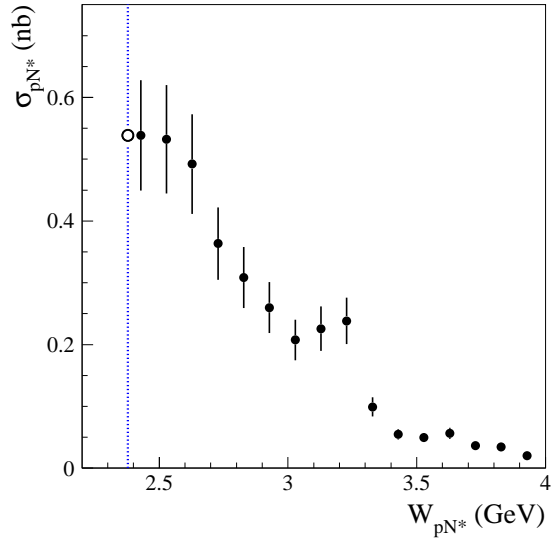
Figure 5: The $e^+e^- \to p\overline{N}(1440) +$ c.c. cross section obtained by *BABAR* (solid circles). The dotted line represents the threshold and the empty circle is the cross section value Coulomb enhanced at threshold, assuming $|G^{pN(1440)}[(M_p + M_{N(1440)})^2]| = 1$.

The cross sections $\sigma_{\Sigma^0\overline{\Sigma^0}}$ and $\sigma_{\Lambda\overline{\Sigma^0}}$ have been measured by the *BABAR* Collaboration for the first time [10], although with very large errors as well as $e^+e^- \to n\overline{n}$ measured by the FENICE Collaboration [11] . The cross sections concerning strange baryons are consistent with U-spin invariance expectation [12], obtained under the assumption of negligible electromagnetic transitions between U-spin triplet and singlet [3].

Remnants of Coulomb interactions at the quark level have been assumed to explain non

vanishing cross sections at threshold in the case of neutral baryon pair production.

However, vanishing cross sections at threshold, rising according to the baryon velocity phase space factor, cannot be excluded according to the present $BABAR$ data, as shown in fig. 6. A much better accuracy is needed to settle this issue.

In conclusion, in the case of $e^+e^- \to p\overline{p}$ near threshold as measured by $BABAR$ it has been shown that, while at threshold there is the expected Coulomb enhancement factor in agreement with the non vanishing cross section, but $|G^p(4M_p^2)| = 1$, above threshold there is no resummation factor. Other charged baryon pair cross sections $e^+e^- \to \mathcal{B}\overline{\mathcal{B}}$ show near threshold a similar behavior, within the errors.
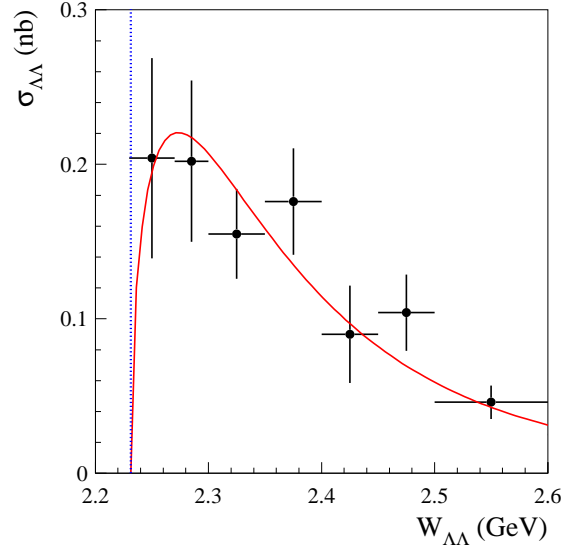
Figure 6: The $\Lambda\overline{\Lambda}$ cross section obtained by $BABAR$. The dotted line represents the threshold and a fit is reported, obtained including the outgoing baryon velocity factor and by means of an effective FF behaving like a dipole.

New data near threshold are coming from CMD2 and SND at VEPP2000 and on a larger interval from BESIII at BEPCII by means of initial state radiation, as well as a test of the Sommerfeld rescattering formula on a pure QED process, like $e^+e^- \to \tau^+\tau^-$.

The investigation of the time-like behavior of nucleon FF's has been carried out by many authors using different approaches, models and phenomenological descriptions; in Ref. [13] we report only an incomplete list. However, the result we present in this letter is a pure statement of fact and hence completely model-independent. Possible interpretations and phenomenological explanations are under study [14].

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who achieved most of the experimental results.

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